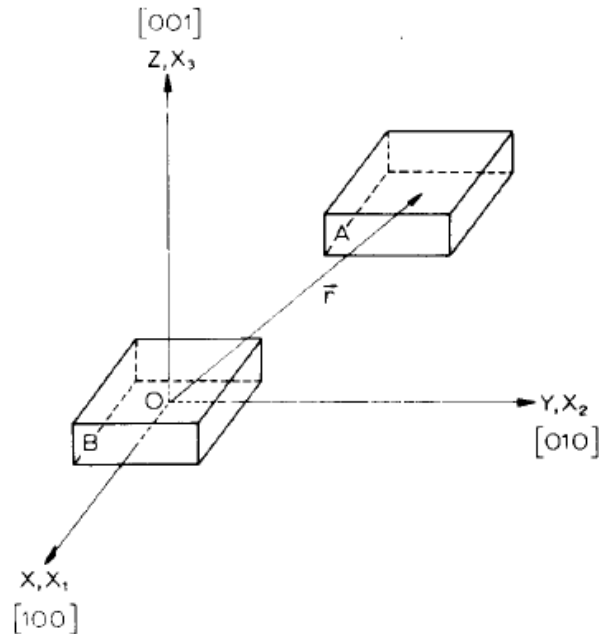


Ref. 133 Elastic strain energy and interactions



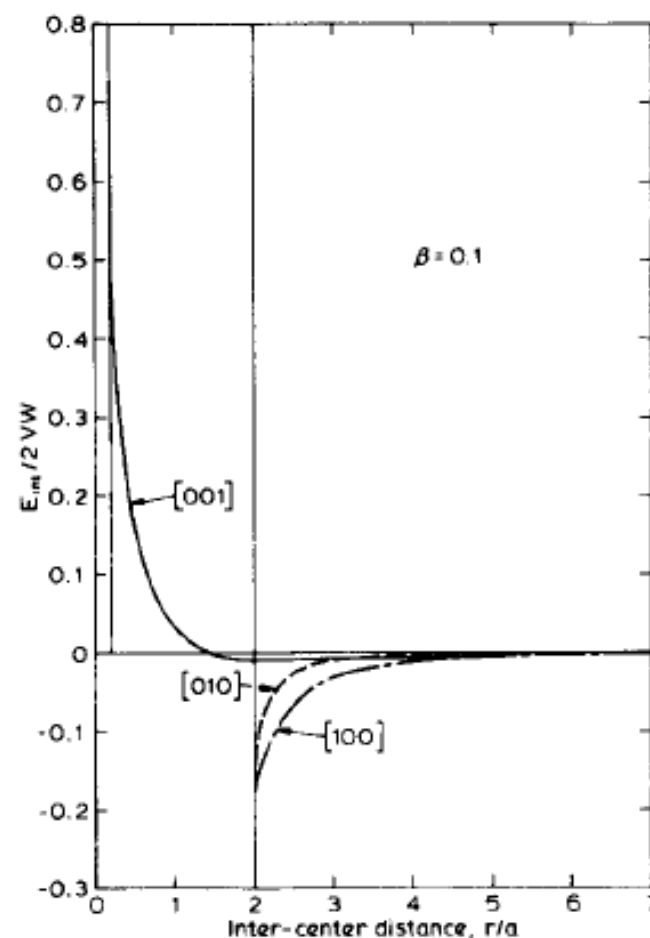
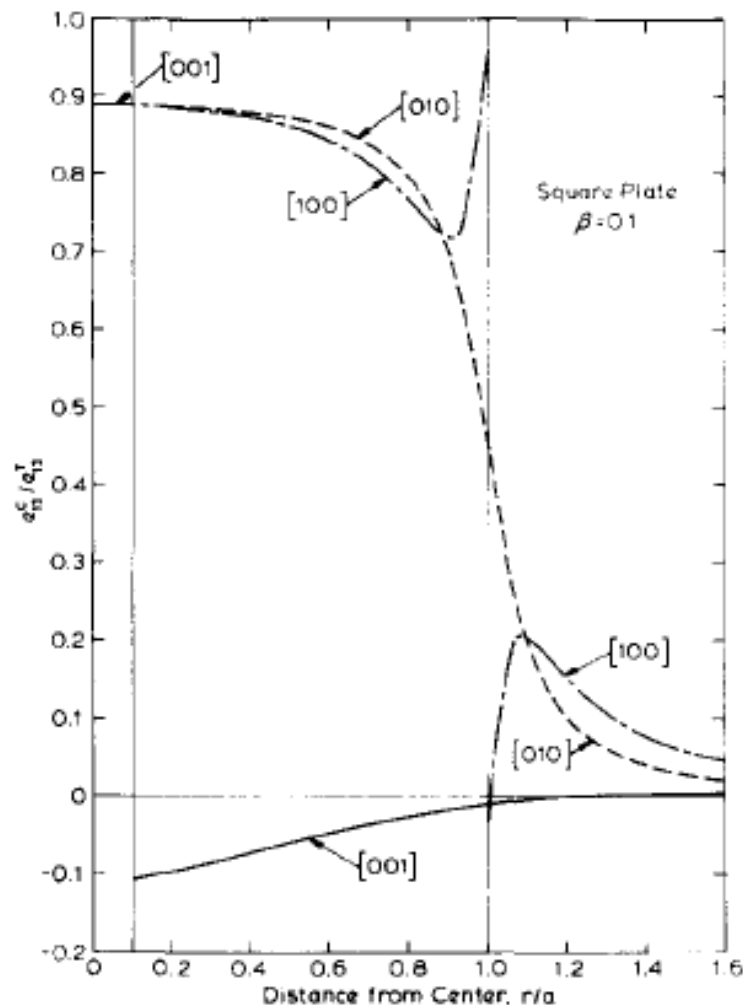
$e_{13}^T = e_{31}^T \neq 0$
all other components are zero

Elastic strain energy

$$W = \frac{1}{2V} \iiint_V (P_{ij}^T - P_{ij}^C(\vec{r}')) e_{ij}^T dV(\vec{r}')$$

Interaction energy

$$E_{int}(\vec{r}) = - \iiint_{V_A} P_{ij}^C(B) e_{ij}^T(A) dV$$



We note that in the interaction energy calculation, the same stress-free transformation strain e_{ij}^T is assumed for both plates. Hence, if the centers of the two square plates are aligned along the $[001]$ direction, and their transformation strains are simple shears of opposite sign in the x - z plane, the magnitudes of their elastic interaction energies are the same but the signs are reversed. Thus, when $\beta=0.1$, the elastic interaction energy can decrease the total (self) shear strain energy of plates aligned in the $[001]$ direction by a factor of 0.48.

Ref. 132 Thermoelastic martensite

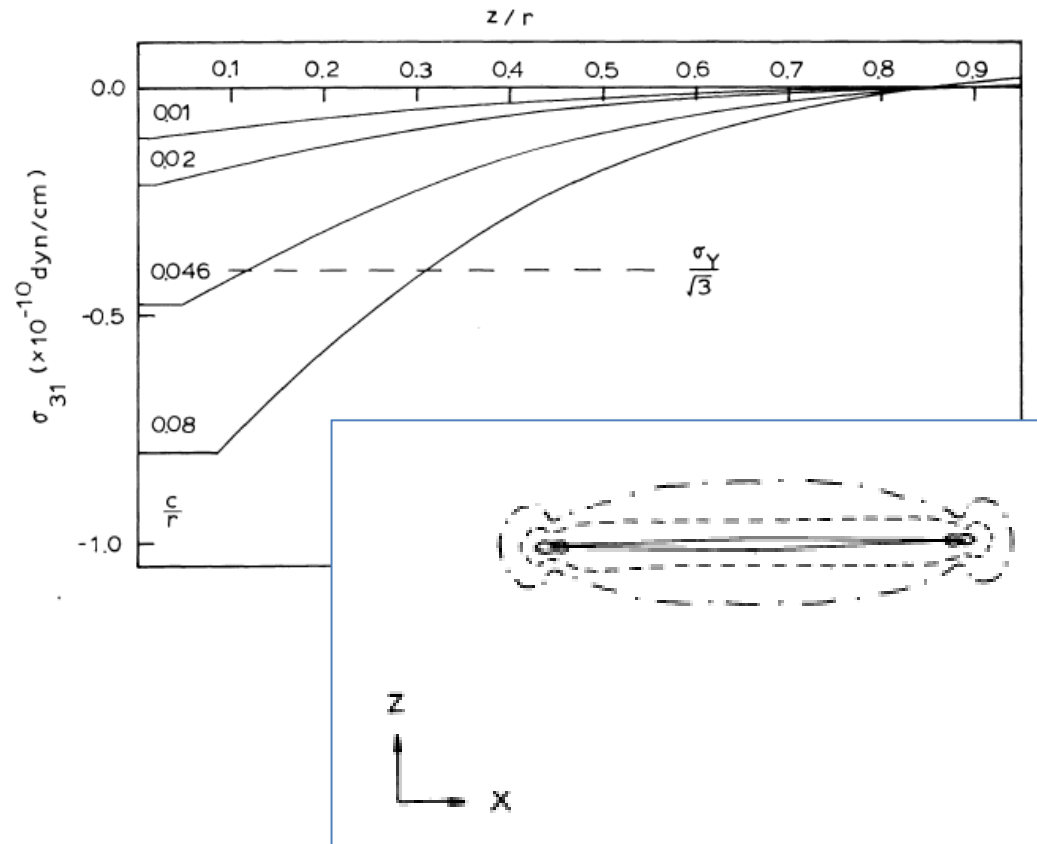
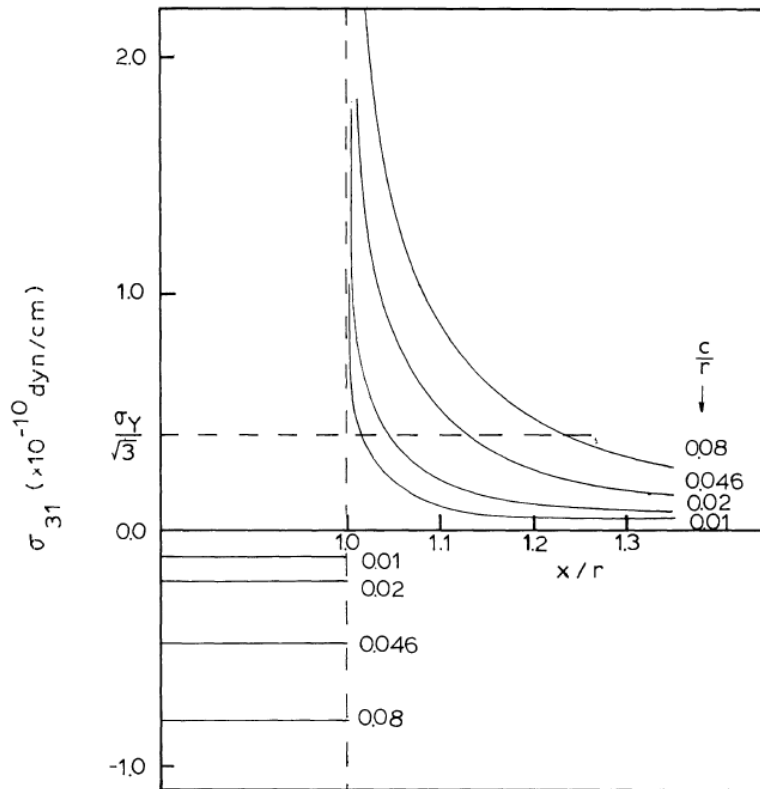
→ Calculation of plastic accommodation area by the transformation of ellipsoidal plate

1. Obtain constrained shear stress σ_{13}^c
2. Use Von mises criterion for yielding

$$\sigma_{\text{eff}}^c \geq \sigma_y$$

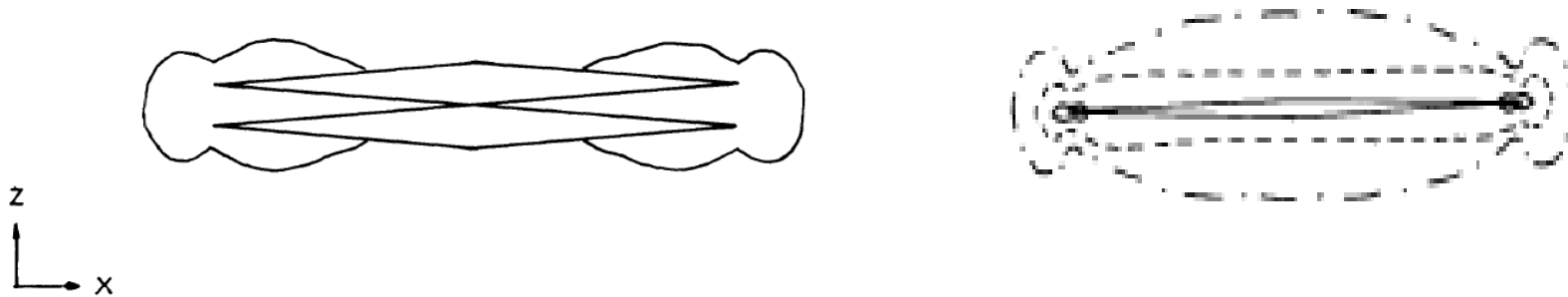
$$\sigma_{\text{eff}}^c = \sqrt{\frac{1}{2}[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2] + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)}$$

For single plate



As shown in Section 5.5.1, the development of σ_{13} stress in the [001] direction in the matrix due to a martensite plate lying in the xy plane with transformation strain in the x direction favors the formation of a second plate of opposite shape strain. If we allow ellipsoidal plates

An assembly of plates with opposite shape strain



- In an assembly, plastic zone diminishes
- minimize the strains accommodation the transformation shape change
- Total strain energy of the system is reduced.

Interpretation

Free surface

Constrained strain = 0

At the tip, the same direction of strain favored

At the broad interface, opposite strain favored

Most of plastic zone exists on the broad interface.

Therefore, the minimization of strain energy can be realized when the broad interface are exposed to constrained strain minimized situation, in other word, free surface.